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ON THE SOLUTION OF DIFFRACTION PROBLEMS FOR PERFECTLY RIGID AND PERFECTLY SOFT WEDGE-SHAPED SCREENS*

V. B. PORUCHIKOV

The problems of diffraction for a perfectly rigid and a perfectly soft screens are well known in acoustics /1—3/. The present paper gives the relations connecting the solutions of these problems for the case of wedge-like screens. In particular, a formula is given which can be used to obtain a solution of the problem of diffraction on a perfectly soft wedge from the solution of the analogous problem for a perfectly rigid wedge.

Let us consider the diffraction of a nonstationary acoustic wave $\varphi = \varphi_0(r, \theta - \theta_0, z, t)$, $0 \leqslant \theta_0 \leqslant \gamma$ on a wedge of opening angle β, formed by two half-planes $\theta = 0$ and $\theta = \alpha$ $(\alpha = 2\pi - \beta)$. The acoustic medium occupies the region $0 < \theta < \alpha$, and the conditions $\varphi = 0$ (first problem) or $\partial\varphi / \partial\theta = 0$ (second problem) are specified on the boundary half-planes $\theta = 0, \alpha$. Here r, θ, z represent a cylindrical coordinate system in which the z-axis coincides with the angle edge and φ denotes the velocity potential. The incident wave $\varphi_0(r, \theta - \theta_0, z, t)$ is assumed to be such, that when $\theta_0 = 0$, then the wave emerges from the region $0 < \theta < \alpha$. In this case a range of values of $\theta_0: 0 \leqslant \theta_0 \leqslant \gamma$ $(\gamma < \alpha)$ exists for which the incident wave in question $\varphi_0(r, \theta - \theta_0, z, t)$ also emerges from the region $0 < \theta < \alpha$. It is precisely for this interval $0 \leqslant \theta_0 \leqslant \gamma$ that the problems of diffraction in question are considered.

If we seek the solutions φ_1 and φ_2 of the first and second problem in the form $\varphi_1 = \Phi_1 + \varphi_0$ and $\varphi_2 = \Phi_2 + \varphi_0$, then we obtain the following systems in terms of the Laplace transforms in t for determining the perturbations $\overline{\Phi}_j(r, \theta, z, p, \theta_0)$ $(j = 1, 2)$:

$$\Delta\overline{\Phi}_1 = p^2\overline{\Phi}_1 \quad (\Delta \equiv \partial^2 / \partial r^2 + r^{-1}\partial / \partial r + r^{-2}\partial^2 / \partial\theta^2 + \partial^2 / \partial z^2) \tag{1}$$
$$\overline{\Phi}_1 = -\overline{\varphi}_0(r, \theta - \theta_0, z, p) \quad (\theta = 0, \alpha), \quad \overline{\Phi}_1 \to 0 \ (r \to \infty), \ r\partial\overline{\Phi}_1 / \partial r \to 0 \ (r \to 0)$$

$$\Delta\overline{\Phi}_2 = p^2\overline{\Phi}_2, \quad \frac{\partial\overline{\Phi}_2}{\partial\theta} = -\frac{\partial}{\partial\theta}\overline{\varphi}_0(r, \theta - \theta_0, z, p) = \frac{\partial}{\partial\theta_0}\overline{\varphi}_0(r, \theta - \theta_0, z, p) \quad (\theta = 0, \alpha), \quad \overline{\Phi}_2 \to 0 \ (r \to \infty), \ r\partial\overline{\Phi}_2 / \partial r \to 0 \ (r \to 0) \tag{2}$$

In (1) and (2) we have

$$\overline{f} = \int_{-\infty}^{\infty} f e^{-pt} dt \quad (f = \varphi_0, \Phi_1, \Phi_2)$$

where $\operatorname{Re} p > 0$ (since $f \equiv 0$ when $t < t_0(r, \theta, z)$) and the speed of sound can be assumed equal to unity without loss of generality. The last conditions in (1) and (2) which are assumed uniform in θ and z, ensure the uniqueness of the solutions of the problems stated /3/ (the condition that $\overline{\Phi}_j \to 0$ as $r \to \infty$ $(j = 1,2)$ excludes, for the time being, the cases of incident plane waves arriving from infinity and giving rise to the waves reflected from the angle edges). Assuming that $\overline{\Phi}_j$ $(j = 1,2)$ are analytic functions of the arguments r, θ, z in the region $0 < \theta < \alpha$, we defferentiate the equation for $\overline{\Phi}_2$ with respect to θ and introduce the function

$$\overline{\Phi}_2' = -\int_0^{\theta_0} \frac{\partial\overline{\Phi}_2}{\partial\theta} d\theta_0$$

Then from (2) we obtain the following system for determining Φ_2' (assuming that the last estimates in (2) are uniform with respect to θ_0 for $0 \leqslant \theta_0 \leqslant \gamma$):

$$\Delta\overline{\Phi}_2' = p^2\overline{\Phi}_2', \quad \overline{\Phi}_2' = -\overline{\varphi}_0(r, \theta - \theta_0, z, p) + \overline{\varphi}_0(r, \theta, z, p) \quad (\theta = 0, \alpha), \quad \overline{\Phi}_2' \to 0 \ (r \to \infty), \ r\partial\overline{\Phi}_2' / \partial r \to 0 \ (r \to 0) \tag{3}$$

Returning to the system (1), we see that the function $\overline{\Phi}_1 - \overline{\Phi}_1|_{\theta_0=0}$ also satisfies the system (3). Therefore, making use of the uniqueness of the solution, we obtain

$$\overline{\Phi}_1 - \overline{\Phi}_1|_{\theta_0=0} = \overline{\Phi}_2'$$

or

$$\overline{\Phi}_1 = -\int_0^{\theta_0} \frac{\partial\overline{\Phi}_2}{\partial\theta} d\theta_0 + \overline{\Phi}_1|_{\theta_0=0} \tag{4}$$

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Differentiating (4) with respect to θ_0 and integrating in θ', we obtain

$$\bar{\Phi}_2 = -\int_0^{\theta} \frac{\partial \bar{\Phi}_1}{\partial \theta_0} d\theta + \bar{\Phi}_2 \Big|_{\theta=0} \tag{5}$$

Similarly, using the fact that the condition $\bar{\Phi}_1 = -\bar{\varphi}_0$ for $\theta = 0, \alpha$ can be replaced, (with help of the equation $\Delta\bar{\Phi}_1 = p^2\bar{\Phi}_1$) by the condition

$$\frac{\partial^2 \bar{\Phi}_1}{\partial \theta^2} = -\frac{\partial^2 \bar{\varphi}_0}{\partial \theta^2} = -\frac{\partial^2 \bar{\varphi}_0}{\partial \theta_0^2} \quad (\theta = 0, \alpha)$$

introducing a new function

$$\bar{\Phi}_1' = -\int_0^{\theta_0} \frac{\partial \bar{\Phi}_1}{\partial \theta} d\theta_0 \tag{6}$$

we obtain, from (1), the following system for determining the function $\bar{\Phi}_1'$:

$$\Delta\bar{\Phi}_1' = p^2\bar{\Phi}_1', \quad \frac{\partial \bar{\Phi}_1'}{\partial \theta} = \frac{\partial \bar{\varphi}_0}{\partial \theta_0} - \frac{\partial \bar{\varphi}_0}{\partial \theta_0}\Big|_{\theta_0=0} \quad (\theta = 0, \alpha), \quad \bar{\Phi}_1' \to 0 \ (r \to \infty), \ r\partial\bar{\Phi}_1' / \partial r \to 0 \ (r \to 0) \tag{7}$$

Returning now to system (2) we see, that the function $\bar{\Phi}_2 - \bar{\Phi}_2|_{\theta_0=0}$ also satisfies the system (7). Therefore, using the fact that the solution of this system is unique, we obtain

$$\bar{\Phi}_1' = \bar{\Phi}_2 - \bar{\Phi}_2|_{\theta_0=0}$$

or

$$\bar{\Phi}_2 = -\int_0^{\theta_0} \frac{\partial \bar{\Phi}_1}{\partial \theta} d\theta_0 + \bar{\Phi}_2 \Big|_{\theta_0=0} \tag{8}$$

Differentiating (8) with respect to θ_0 and integrating in θ, with the boundary conditions taken into account, we obtain

$$\bar{\Phi}_1 = -\int_0^{\theta} \frac{\partial \bar{\Phi}_2}{\partial \theta_0} d\theta - \bar{\varphi}_0 (r, -\theta_0, z, p) \tag{9}$$

We note that although in the course of deriving the formulas (4) — (9) we exluded the cases of diffraction of nonstationary plane waves when the waves reflected from the angle edges were arriving from infinity, the formulas (4) — (9) cover these cases. The cases in question are obtained as limiting cases of the problems of diffraction of the corresponding spherical waves when their sourves are removed to infinity.

When $\theta_0 = 0$, the formulas (5) and (9) yield

$$\bar{\Phi}_2 (r, \theta, z, p) = -\int_0^{\theta} \frac{\partial}{\partial \theta_0} \bar{\Phi}_1 (r, \theta, z, p, \theta_0) \Big|_{\theta_0=0} d\theta + \bar{\Phi}_2 (r, 0, z, p) \tag{10}$$

$$\bar{\Phi}_1 (r, \theta, z, p) = -\int_0^{\theta} \frac{\partial}{\partial \theta_0} \bar{\Phi}_2 (r, \theta, z, \theta_0) \Big|_{\theta_0=0} d\theta - \bar{\varphi}_0 (r, 0, z, p) \tag{11}$$

where $\quad f (r, \theta, z, p) \equiv f (r, \theta, z, p, \theta_0 |_{\theta_0=0} \quad (f = \bar{\Phi}_1, \bar{\Phi}_2, \bar{\varphi}_0)$

Out of the formulas (10) and (11), the latter is more suitable for practical applications. Using (11) we can obtain a solution of the diffraction problem for a perfectly soft wedge-shaped screen from the solution of the same problem for a perfectly rigid, wedge-shaped screen.

Using in (11) the relations $\bar{\Phi}_j = \bar{\varphi}_j - \bar{\varphi}_0 \ (j = 1, 2)$ and applying to both parts of (11) the inverse Laplace transform, we obtain the following formula (in which the derivative $\partial\varphi_2 / \partial\theta_0$ is assumed generalized):

$$\varphi_1 (r, \theta, z, t) = -\int_0^{\theta} \frac{\partial}{\partial \theta_0} \varphi_2 (r, \theta, z, t, \theta_0) \Big|_{\theta_0=0} d\theta \tag{12}$$

It must be noted that the formulas (11) and (12) remain valid in the case of stationary diffraction problems when the time dependence in (12) is determined by the factor $\exp (ikt)$, $\mathrm{Im} k = 0$ (the substitution $p = ik$ should also be made in (11)). Thus we have proved the following theorem.

Let an arbitrary acoustic wave $\varphi_0 (r, \theta, z, t)$ arrive from the region $0 < \theta < \alpha$ to impinge on a wedge of angle β formed by the half-planes $\theta = 0$ and $\theta = \alpha \ (\alpha = 2\pi - \beta)$. Then the solution $\varphi_1 (r, \theta, z, t)$ of the problem of diffraction of the wave in question on a perfectly soft wedge is given in terms of the solution $\varphi_2 (r, \theta, z, t, \theta_0)$ of the problem of diffraction, on a perfectly rigid wedge, of an incident wave with a displacement $\varphi_0 (r, \theta - \theta_0, z, t)$ according to formula (12) (or for the transforms of the perturbations $\overline{\Phi}_j = \overline{\varphi}_j - \overline{\varphi}_0$, by formula (11) in which the substitution $p = ik$ should be made in the stationary case).

As a corollary of the above theorem we obtain from (12)

$$\int_0^\alpha \frac{\partial}{\partial \theta_0} \varphi_2(r, \theta, z, t, \theta_0)\Big|_{\theta_0=0} d\theta = 0 \tag{13}$$

hence it follows that (12) can be written in the form

$$\varphi_1(r, \theta, z, t) = \int_\theta^\alpha \frac{\partial}{\partial \theta_0} \varphi_2(r, \theta, z, t, \theta_0)\Big|_{\theta_0=0} d\theta \tag{14}$$

We also note that in the particular case of φ_0 being independent of θ (this can only take place when the wave φ_0 appears at the wedge edge), the solution φ_2 is independent of θ_0 and $\partial\varphi_2 / \partial\theta_0 \equiv 0$. In this case an additional perturbation Φ_1 extinguishes the incident wave and we have $\varphi_1 \equiv 0$. Thus in this particular case the left and right hand parts of (12) and (14) vanish identically.

In conclusion we note, that it can be verified that the known solutions of the stationary and nonstationary problems of diffraction of the plane, spherical and cylindrical waves on an angle /1,3—5/ satisfy the relations (11)—(14).

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